

INDIAN STATISTICAL INSTITUTE, Bangalore Centre
Midsem
September 7, 2026

Course Name: Real Analysis

Duration: 2 hours

1. Determine whether the following statements are TRUE or FALSE with justification.

[a] Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be two functions. Then if $g \circ f$ is injective, then f is injective. [2]

[b] The set of algebraic numbers is uncountable. Note that algebraic numbers are the roots of polynomials in one variable having integer coefficients. [3]

[c] Let $A, B \subseteq (0, \infty)$ be non-empty and bounded sets. Then

$$\sup(AB) = (\sup A)(\sup B),$$

where $AB = \{ab : a \in A, b \in B\}$. [3]

[d] There exists no bounded sequence with infinitely many convergent subsequences. [3]

[e] Let $\{a_n\}$ be a bounded sequence and let $L \in \mathbb{R}$. If every subsequence of $\{a_n\}$ has a further subsequence converging to L . Then $a_n \rightarrow L$, as $n \rightarrow \infty$. [3]

2. Let $A \subseteq \mathbb{R}$ be non-empty and bounded above.

[a] Prove that there exists a sequence $\{a_n\} \subseteq A$ such that

$$a_n > \sup A - \frac{1}{n} \text{ and } a_n \rightarrow \sup A.$$

[2]

[b] Let $s \in \mathbb{R}$ satisfies the following: for every $\epsilon > 0$, there exist $a, b \in A$ such that

$$s - \epsilon < a \leq s \leq b + \epsilon.$$

Does this imply $s = \sup A$? [4]

[c] For each $a \in A$, define

$$B_a = \{x \in A : x < a\}.$$

Let $S = \{\sup B_a : a \in A \text{ and } B_a \neq \emptyset\}$. Determine $\sup S$. [6]

3. [a] Let $\{a_n\}$ be a bounded sequence such that $\{a_{2n}\}$ and $\{a_{2n+1}\}$ converges to the same limit L . Then prove that $a_n \rightarrow L$. [3]

[b] Let

$$a_1 = 1 \text{ and } a_{n+1} = \frac{a_n + 2}{a_n + 1}.$$

Determine whether the sequence $\{a_n\}$ converges. If $\{a_n\}$ converges then find the limit. [6]

[c] Let

$$a_n = \sin\left(\frac{n\pi}{2}\right) + \frac{1}{n}$$

Find $\limsup a_n$ and $\liminf a_n$. [5]